Burning the Cosmic Commons: Evolutionary Strategies for Interstellar Colonization

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Abstract

Attempts to model interstellar colonization may seem hopelessly compromised by uncertainties regarding the technologies and preferences of advanced civilizations. If light speed limits travel speeds, however, then a selection effect may eventually determine frontier behavior. Making weak assumptions about colonization technology, we use this selection effect to predict colonists' behavior, including which oases they colonize, how long they stay there, how many seeds they then launch, how fast and far those seeds fly, and how behavior changes with increasing congestion. This colonization model explains several astrophysical puzzles, predicting lone oases like ours, amid large quiet regions with vast unused resources.

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1 Economics and the Search for Extraterrestrial Intelligence

For many decades, the search for extraterrestrial intelligence (ETI) has proceeded using advanced astronomical techniques (Cosmovici, Bowyer, & Werthimer, 1997). In contrast, the social theory which directs such empirical attentions, suggesting where such ETI may be distributed and the kinds of observable activities it may be engaged in, has been relatively primitive.

Most research in this area has been done by astronomers, who have given careful theoretical attention to the distribution of planets like ours near stars like ours. Sophisticated biological theory has also been applied to the question of where life such as we see on Earth could arise, travel, and survive (Lawler, 1998). Some attention has even been given to how often simple life may give birth to a technologically advanced civilization like humanity's.

Regarding the possible activities of advanced civilizations, sophisticated engineering has been applied to place bounds on the feasibility and costs of interstellar travel (Crawford, 1995) and on large visible engineering projects such as intercepting most of the light of a star, or even disassembling planets and stars (Criswell, 1985). Sophisticated communication theory and astrophysics has also been applied to suggest low cost methods for an ETI to communicate across astronomical distances.

However, when it comes to considering which physically possible activities advanced civilizations might choose to engage in, the search for ETI has been mostly informed by astronomer's intuitive theories of the nature of societies. (There are a few notable exceptions (Finney, 1985; Wachter, 1985; Bainbridge, 1984).)

For example, a striking empirical puzzle regarding ETIs is the lack of alien visitors here on Earth (Brin, 1983), even though ETI does not seem terribly hard to evolve, the universe is very big, and ETI civilizations could have been expanding for billions of years. Attempts at explanations include our star system being part of an isolated zoo (Ball, 1973), or being within an empty border zone between civilizations (Turner, 1985).

Also, while simple population diffusion processes have been frequently used to model the expansion of ETI civilizations (Jones, 1976, 1995; Newman & Sagan, 1981; Bainbridge, 1984; Fogg, 1987), crucial features of such models are typically based on simple intuitive arguments. For example, some feel it is the nature of life to fill all available niches, while others see advanced civilizations as having overcome base expansionist tendencies.

Those who lean toward niche-filling tend to assume fast cheap interstellar travel, high per-capita wealth, and population growth and emigration rates no less than that of recent human history (Jones, 1995; Barrow & Tipler, 1986). This implies fast expansion rates for ETI civilizations, and hence suggests that the lack of alien visitors here now implies that ETI is very rare.

Others, including the most prominent astronomers who discuss ETI, tend to see interstellar travel as slow and expensive, per-capita wealth as limited (Drake & Sobel, 1992), and recent human population trends as historical abberations (Newman & Sagan, 1981). Such assumptions can imply slow expansion rates, allowing our galaxy to be full of ETIs while Earth is untouched. This can support optimism in the search for ETI. For example, Harvard astronomer Michael Papagiannis argues that

> The limits of growth in a finite system, which will be imposed on all stellar civilizations by the colossal distances that separate the stars, will affect the natural selection of these civilizations. Those that manage to overcome their innate tendencies toward continuous material growth and replace them with non-material goals will be the only ones to survive this crisis. As a result the entire Galaxy in a cosmically short period will become populated by stable, highly ethical and spiritual civilizations. (Papagiannis, 1984)

Astronomers Carl Sagan and William Newman are even willing to assume zero-population growth, which implies a very slow rate of expansion (Newman & Sagan, 1981). They reason

> Those civilizations devoted to territoriality and aggression and violent settlement of disputes do not long survive after the development of apocalyptic weapons. Civilizations that do not self-destruct are pre-adapted to live with other groups in mutual respect. This adaptation must apply not only to the average state or individual, but, with very high precision, to every state and every individual within the civilization ... the result is that the only societies long-lived enough to perform significant colonization of the Galaxy are precisely those least likely to engage in aggressive galactic imperialism. (Sagan & Newman, 1983)

Since the search for ETI seems to be an extreme version of the proverbial search for a needle in a haystack, theoretical expectations are important in guiding empirical searches for ETI. And since the largest theoretical uncertainties seem to regard the social aspects of ETI, it seems desirable to replace astronomer's intuitive social theories with more sophisticated social theory regarding ETI. In particular, formal economic theory may be both general enough to hope to apply to the extreme circumstance of ETI societies, and formal enough to allow a clearer identification of which assumptions lead to which conclusions.

Economic theory has been applied to the consideration of human institutions and policy regarding space for a while now (Banks, Ledyard, & Porter, 1989; Butler & Doherty, 1991); perhaps it is time to consider a wider scope for economics in space studies. An economic analysis of interstellar colonization might not only inform the search for ETI, it may also inform us about the possible futures for our distant descendants, and may serve as a colorful base for generalizations to analogous spatial colonization processes. Such analogous processes include colonization by humans in the recent (Ely, 1918; Luthy, 1961) or remote past, by other biological organisms both large and small, and by agents colonizing more abstract spaces, such as companies colonizing spaces of possible concepts and designs.

2 Overview of Results

This paper offers an economic style analysis of the consequences of a wave of colonization having expanded from some source for long enough, and in a decentralized enough manor, that a selection effect determines frontier behavior. That is, we assume sufficient diversity or mutability of colonist values, sufficient autonomy for colonies to express those preferences in differing colonization strategies, and sufficient spacing between the origins of colonization waves to allow the leading edge of colonization to come to be dominated by a selection effect: leading edge colonists must travel at the maximum sustainable average speed.

While these are not the only reasonable assumptions one could make, they are among those commonly considered, and this paper describes some relatively general implications of these assumptions. In contrast to the population-biology based models previously used to discuss ETI expansion, which implicitly make specific behavioral assumptions such as specific crude correlations between emigration actions, local population levels, and the motion of a colonization wave (Murray, 1993; Newman, 1980, 1983), we here explicitly consider strategic actors who may choose from a wide range of behaviors.

Specifically, we consider a wave of colonization, wherein seeds land at oases, grow colonies there, and then send out more seeds, each of which travels much far than the oasis spacing. We assume that property rights in virgin oases are not enforced, so oases go to whoever arrives first. (Property rights seem especially hard to enforce regarding leading-edge seeds traveling at a substantial fraction of the speed of light.) We also assume that this process has persisted long enough that frontier behavior is dominated by a selection effect: there are no other behaviors which would allow a population of seeds to sustain or grow their number while pulling ahead of the overall colonization wave. (Selection effects have been observed in simulations of interstellar colonization across even relatively small scales (Bainbridge, 1984), and have been considered as explanations for preferences and behaviors in a wide variety of economic contexts (Hansson & Stuart, 1990; Blume & Easley, 1992).)

In this analysis, we allow the behavior of a seed or colony to be contingent on its location in space-time. As random factors make colonists fall back or jump ahead relative to the overall colonization wave, colonists adjust their behavior in light of their preferences regarding creating descendants at various points in space-time, and their expectations about the equilibrium behavior of other colonies. Colonists are free to choose the types of oases they land at, how long they stay there, the speed, hardness, and perception abilities of seeds, and how far those seeds fly before they attempt to colonize a new oasis. Assuming all colonies are risk-neutral and have the same preferences and technology, we derive constraints on such equilibrium preferences and strategies.

Since eventually most of the volume of the colonized universe should be far behind the colonization frontier, we are particularly interested in the preferences and behavior of colonists who have fallen far behind the frontier, perhaps left behind at an oasis after its main seed exodus.

We consider first the behavior at the very leading edge of colonization, then consider behaviors elsewhere in the colonization process. We then consider behaviors in a traveling wave of colonization, where far from the colonization origin the process has settled down to a stable one-dimensional pattern moving through space. Finally, using particular functional forms, we obtain closed form expressions for a smooth approximation, and we discuss how these results generalize when there are a diversity of oasis types. We get the following results.

At the leading edge, great growth at oases is matched by great death between oases; on average only one of all the seeds launched from an oasis survives to colonize a new oasis. Also, a great premium is placed on seed hardness; a trillion plain seeds are worth as much as a million seeds twice as penetrating, which are worth as much as a thousand seeds which penetrate twice as much again.

Everywhere in the colonization wave, succeeding generations stay longer at oases before sending out seeds, and place a smaller premium on being closer to the leading edge. A large premium is everywhere placed on seed hardness, as the ideal elasticity between seed cost and seed hardness is at least the total seed mortality factor.

As congestion increases, making it difficult for seeds to find unoccupied oases, seeds attempt landings at more and more oasis types. Even with diverse oasis types, however, all seeds launched from a given space-time region have the same speed and hardness, and target the same destination space-time region. Comparing nearby oases, there is some cutoff growth rate such that oases with that cutoff rate launch seeds, and colonies which are growing faster wait longer. If seed launching reduces the local growth rate, colonists left behind after a seed exodus will want to leave as soon as possible.

In a traveling wave of colonization, individual seeds always fly faster than the speed of the colonization wave, and colonists have infinite patience regarding investments whose returns keep up with that traveling wave. Succeeding generations of colonies fall further behind the leading edge, increase in number, and per colony deliver more seeds to each succeeding space-time region. With finite oases and a minimum mortality rate for seeds in flight, there is an upper limit to the fraction of oases which get colonized, and a lower limit to the oasis growth rate colonists will tolerate. When seed hardness is fixed, each generation sends out more seeds at a slower speed, which have a lower mortality rate at the end of their flight.

If the preferences of the very first colonists were the preferences that would eventually be selected for, or if early colonists directly valued having their descendants selected for, this sort of analysis might inform the strategies of the earliest colonists, perhaps including when our descendants should leave our solar system. We find two cases where we can show that seeds should not be launched until after local growth rates slow down: anywhere in a traveling wave, and at any generalized leading edge when seeds suffer a constant mortality between oases. This suggests that selected-preference colonists will not leave our solar system until after economic growth rates reverse their historical trend so far toward increasing growth rates.

Regarding the possibility that an ETI colonization wave has already passed this way, we can compare this theoretical analysis with astrophysical observations now considered relevant to the search for ETI. This includes the "Great Silence" (Brin, 1983); no alien ETI appear on Earth now, no strong radio signals are being steadily sent here from nearby stars, and astrophysicists have had a great and increasing success at explaining astrophysical phenomena in terms of simple physical processes, without reference to ETI societies. For example, we can see that any civilizations surrounding nearby stars dump little nuclear waste into their stars (Whitmire & Wright, 1980), don't use powerful radars (Tarter, 1997), don't use certain forms of starship drives (Drake & Sobel, 1992), and intercept less than one percent of their star's light (Jugaku & Nishimura, 1997).

What does the model in this paper suggest a long-colonized region should look like? Surprisingly, this model allows for lone oases like ours amid large regions apparently devoid of advanced activity, and containing large amounts of unused resources. Very few colonists should remain far behind the frontier, and any stragglers should be in a big hurry to catch up with the leading edge, rather than to communicate with us, or to take on slow resource extraction projects which might result in large visible structures.

On the other hand, the model also suggests that we should a wall of colonization at some, perhaps large, distance from here. And we should observe an cutoff in the distribution of growth rates that observed resources can support, with an absence of resources supporting rapid growth that simple physical processes led one to expect to be more common. We may also observe garbage artifacts where the missing oases once were, and the remains and exhaust of seeds. Finally, the model offers no obvious reason for such colonists to take any trouble to hide their activities from us or anyone else. More work seems required to see if this model is consist with our astrophysical observations.

3 Assumptions and Notation

Think of space as mostly nearly-empty space with a few exceptional "oases" such as stars. The resources available to a colony at an oasis depend on how long that colony has been growing there. Assuming there is effectively only one interesting resource dimension, and let the increasing function $\tilde{R}(s)$ describe the total resources available to a colony which as been growing at an oasis for a duration s. (Section 15 summarizes all our notation.)

Note that this form $\tilde{R}(s)$ implicitly assumes that oases are identical across space and time, and that technology is unchanging. We thus neglect the diversity of star types, and the overall evolution of the universe. Oasis diversity will be examined in section 12. Unchanging technology seems a plausible assumption in the limit when physical laws become well-known and a very wide space of technological designs becomes well explored.

Resources are required to launch a new seed with speed v and hardness h. Assume that all seeds launched from a given oasis have the same speed v and hardness h. (This assumption will follow from an assumption of risk-neutrality made in section 5. Let $F(v, h)$ be the fixed resource cost to build a seed launching facility, and let $C(v, h) \geq 1$ be the constant marginal cost to launch one more seed. If all seeds launched from an oasis are launched at the same time, then $N = (\tilde{R}(s) - F)/C$ is the number of seeds launched per oasis. (Assumptions which imply identical seed launch dates are given in section 11.)

Seeds traveling between oases may fail to reach their destination due to internal failures, or due to collisions with dust grains or cosmic rays. Let mortality $A(x, v, h) \geq 1$ be the number of seeds with speed v and hardness h which would have to be launched to expect one to survive on average after traveling a distance x. (Note that this form assumes a uniform unchanging inter-oasis medium, neglecting for example the clumping of dust around galaxies.) An noteworthy reference case is *constant mortality*, where A is exponential in x, so there is a constant probability per unit distance of a still-surviving seed being destroyed.

Let r, t denote a space-time event at a radius r and time t from the center and beginning of a spherically-symmetric colonization process. For $\underline{x}$ the minimum distance seeds travel between oases, and n the number density of oases, if $\underline{x} \gg n^{-1/3}$ then we can ignore the detailed locations of oases and just consider $P(r, t)$, the fraction of oases near r that are occupied at time t. The rate at which oases are becoming occupied is $P_t(r, t) \geq 0$.

Assume that at some point a seed must commit to a choice of an oasis it will attempt to colonize, and that if this attempt fails, the seed is too weak to make a second attempt at another oasis. Assume further that seeds have imperfect information about which oases are currently occupied, and about the existence and intentions of other seeds in flight. Since physical limits on travel speeds make it hard to see how one could enforce property rights in virgin oases, we assume that the first seed to land at an oasis gets it, and we neglect any variations in costs to defend oases against new seeds.

Given these assumptions we can focus on the local probability of a successful landing

$$Q(r,t;v,h) = q(P(r,t), P_t(r,t); v, h),$$

where $q(P, P_t; v, h)$ gives the probability of landing at an unoccupied oasis when a fraction P of oases are occupied, when occupancy is increasing at a rate P_t, and when the seed has speed v and hardness h. A specific binary-signal model for $q(P, P_t; v, h)$ will be described in section 8.

Assume that, aside from the problems of in-flight mortality and already occupied oases, seeds can reliably detect and land at suitable oases. Then since it is always possible to randomly choose target oases, we must have $Q \geq 1 - P$. We also assume that $P = 1$ implies $Q = 0$, since one cannot colonize if all oases are taken. We will also typically want to assume *increasing congestion*, with $Q_t \geq 0$ and $Q_r \leq 0$.

The distance between generations of colonies is x, the distance a seed travels, and the time between generations is

$$T = s + \delta(v,h) + x/v,$$

where s is the time between oasis colonization and seed launch, v is seed velocity, and $\delta(v,h)$ is the time delay required for a seed to stop at a new oasis.

Given that one seed successfully colonizes an oasis at r, t, the expected number of seeds which can be delivered to the event $r + x$, $t + T$ can be written as generational *growth*

$$G(x,T;v,h) = \frac{N}{A} = \frac{\tilde{R}(T - \delta(v,h) - x/v) - F(v,h)}{C(v,h)\, A(x,v,h)}.$$

The expected number of successfully colonized oases at $r + x$, $t + T$, however, is

$$G(x,T;v,h)\ Q(r+x,\, t+T; v,h).$$

To simplify our model, we will from here on neglect the dependence of delay, fixed cost, and oasis detection on v and h. That is, we assume $\delta_v = \delta_h = F_v = F_h = q_v = q_h = 0$. Thus we can define an increasing net resource function $R(T - x/v) = \tilde{R}(T - \delta - x/v) - F$. We will also assume $A(x,v,h) = A(b(v,h)x)$, for A an increasing function, so that the seed mortality function retains its shape as seed velocity and hardness varies. Thus we write

$$G(x,T;v,h) = \frac{R(T-x/v)}{C(v,h)\,A(b(v,h)x)} \tag{1}$$

and we subsume δ in s, writing $s = T - x/v$.

For any function $f(z)$, let us write $\ln f(z) = \ln(f(z))$. Then $\ln G$ is locally convex in x, T if and only if $\ln R$ is concave and $\ln A$ is convex (i.e., $\ln A'' \geq 0 \geq \ln R''$), so that the in-flight mortality rate $\ln A'$ increases with distance and the oasis growth rate $\ln R'$ decreases with time. Note that even with a concave total resource growth function $\ln\tilde{R}$, the delay δ and fixed cost F terms can induce non-concave regions in the net resource function $\ln R(s) = \ln\tilde{R}(x-\delta) - F$, where $\ln R'' > 0$. We will typically assume, however, that $\ln R$ is concave at the actual chosen s.

4 At The Leading Edge

At the very leading edge of colonization, $P(r,t)$ approaches zero, and so $Q(r,t)$ must approach one. Assume for the moment that it were regularly possible for the descendants of a seed to grow in number while remaining along some candidate "leading edge", so that for r, t and $r+x$, $t+T$ both "at" the leading edge, $G(x,T;v,h) > 1$. Then it would be possible for seeds at this leading edge to grow a little slower while pulling ahead of this leading edge, via $G(x, T-\epsilon; v,h) > 1$ or $G(x+\epsilon, T; v, h) > 1$. If any "leading edge" colonists took this strategy and did not die out completely, then eventually their descendants should become numerous enough that $P(r,t)$ would not approach zero at the candidate "leading edge". And so it would not actually be the leading edge.

If a leading edge is to be stable against this sort of selection effect, it must not be possible to grow there regularly. That is, the strategies x, T, v, h at a *stable leading edge* must satisfy

$$\max_{x,T,v,h} \frac{x}{T} \quad \text{such that} \quad G(x,T;v,h) \geq \gamma, \tag{2}$$

for $\gamma = 1$. Call the value of this maximum speed w, and call the arguments which produce this maximum $\tilde{x}, \tilde{T}, \tilde{v}, \tilde{h}$. (Naturally, $\tilde{s} = \tilde{T} - \tilde{x}/\tilde{v}$ also.)

If the constraint $G \geq 1$ binds, we have $G = 1$ or $A = N$. This is our first noteworthy result.

Result 1 *At a stable leading edge, on average only one of all the seeds launched from an oasis survives to colonize a new oasis.*

For any γ we can write the FOC (first order condition) for equation 2 with respect to x, T as

$$w = \frac{\tilde{x}}{\tilde{T}} = -\frac{\ln G_T}{\ln G_x} \tag{3}$$

and with respect to v, h as $0 = G_v = G_h$.

Plugging in our expression for G (equation 1), the FOC with respect to x, T becomes

$$\frac{\tilde{s}}{\tilde{x}} = \frac{b\ln A'}{\ln R'}, \tag{4}$$

and the SOC (second order condition) becomes

$$\frac{\ln R''}{\ln R'^2} \leq \frac{\ln A''}{\ln A'^2}.$$

Note that this SOC implies that for constant mortality, where $\ln A'' = 0$, we must have $\ln R'' < 0$.

Result 2 *With constant mortality, leading edge colonists don't launch seeds until after local net growth rates start to slow down.*

To the extent that the earliest colonization efforts, near $r = t = 0$, can be thought of as roughly following leading edge strategies, satisfying equation 2 for some value of γ, under conditions of near constant seed mortality, then colonists should not leave the colonization origin (such as our solar system) in earnest until growth rates begin to slow down there.

Constant mortality is also a useful reference case for considering seed hardness h. With constant mortality, a doubling of seed hardness via $b \to b/2$ makes $A \to \sqrt{A}$, for which one is willing to pay up to $C \to C\sqrt{A}$ when attempting to $\min C\,A$. That is, for $A = N$ one is willing to raise seed costs so much that $N \to \sqrt{N}$, holding and R constant.

Result 3 *With constant mortality at the leading edge, a trillion plain seeds are worth a million seeds twice as penetrating, which are worth a thousand seeds which penetrate twice as much again.*

Note that the same value ratios apply to any innovation which everywhere halves local mortality rates $d\ln A/dx$, since that also makes total $A \to \sqrt{A}$.

5 Behind The Leading Edge

The finite oasis spacing, together with small variations in seed launch times and speeds, should induce noise regarding the seed-landing events $r + x$, $t + T$ of the children of a seed which landed at x, T. The accumulation of such noise will eventually cause some leading edge seeds to fall behind the leading edge of colonization, moving into space-time regions with substantial population $P > 0$ and congestion $Q < 1$. Since eventually most the volume of the colonized universe is in regions occupied by fallen-behind colonists, we are particularly interested in the strategies and preferences of such colonists.

To find such strategies and preferences, we will assume that the induced value $U(r,t)$ to colonists of having a seed successfully land at an oasis at event r,t is given by a *strategy* equation

$$U(r,t) = \max_{x,T,v,h} G(x,T;v,h)\, Q(r+x,\, t+T)\, U(r+x,\, t+T), \tag{5}$$

the expected number of child seeds which successfully land at unoccupied oases near the new event, times the value of having seeds at that new event.

Note that this form assumes risk-neutral colonists and neglects to explicitly consider noise regarding the landing event $r+x,\ t+T$. The reasonableness of these approximations are discussed in sections 9 and 10. Note also that risk-neutrality by itself implies that it is always optimal for all the seeds launched from a given oasis to have the same strategy x,T,v,h.

Let $x(r,t), T(r,t), v(r,t), h(r,t)$ be the arguments which maximize the strategy equation (5), and for any function $f(r,t)$ of space-time, let us write $\bar{f}(r,t) = f(r + x(r,t), t + T(r,t))$, the function f applied to the succeeding generation after r,t. The strategy equation (5) then implies

$$U(r,t) = G(x,T;v,h)\bar{Q}(r,t)\bar{U}(r,t). \tag{6}$$

The local growth rate of the colonist population is given by a *flow* equation

$$\bar{P}_t(r,t) = P_t(r,t)\, G(x,T;v,h)\, \bar{Q}(r,t) \left(\frac{r}{r+x}\right)^2 \left|\det \frac{\partial(r+x,\, t+T)}{\partial(r,t)}\right|^{-1}, \tag{7}$$

where the last two terms correct for the divergence across space-time of colonist strategies. Note that the form of this equation implicitly assumes a uniform distribution of oases.

The strategy and flow equations (5 and 7) are the primary equations of our model, being sufficient for example to numerically compute a colonization equilibrium. It turns out, however, that almost all our results will come from considering the strategy equation alone.

The FOC of the strategy equation (5) with respect to x,T, together with expressions for $\ln U_r, \ln U_t$, imply

$$\ln U_r = \quad -\ln G_x = \quad -\ln \bar{G}_x + \ln \bar{Q}_r, \tag{8}$$

$$\ln U_t = \quad -\ln G_T = \quad -\ln \bar{G}_T + \ln \bar{Q}_t. \tag{9}$$

Since $G_T \geq 0 \geq G_x$, colonists discount returns that clearly fall behind the colonization wave, with $U_r \geq 0 \geq U_t$. If we also assume increasing congestion, $Q_r \geq 0$, $Q_t \leq 0$, then $-\ln G_x \geq -\ln \bar{G}_x \geq 0$ and $\ln G_T \geq \ln \bar{G}_T \geq 0$, so that $\ln U_r$, and $-\ln U_t$ are positive and decreasing with successive generations.

Result 4 *The premium placed on being closer to the leading edge decreases with succeeding generations.*

Substituting for G via equation 1, we find $\ln G_T = \ln R'$, so equation 9 and $\ln R'' \leq 0$ imply $\bar{s} \geq s$.

Result 5 *Succeeding generations stay longer at oases before sending out seeds.*

The FOC for our strategy equation (5) with respect to v, h are $0 = G_v = G_h$. The maximization with respect to hardness h is equivalent to $\min_h C\, A(bx)$. The FOC for this can be written in terms of the optimal elasticity between seed cost and seed hardness as

$$-\frac{\ln C_h}{\ln b_h} = bx \ln A'(bx). \tag{10}$$

The right-hand side of equation 10 is the number of probes that would have to be launched for just one to survive had the ending mortality rate applied over the whole distance. For an increasing mortality rate, $\ln A'' > 0$, this is greater than the total mortality factor A. Thus the optimal elasticity between seed cost and hardness can be very large.

Result 6 *The ideal elasticity between seed cost and seed hardness is at least the total seed mortality factor.*

The FOC for speed v is

$$\ln C_v / x = \ln R' / v^2 - b_v \ln A'. \tag{11}$$

6 A Traveling Wave

The differential equations which are typically used to describe spatial growth and combustion for physical and biological systems in uniform media typically have traveling wave solutions (Murray, 1993), and typically a wide range of initial conditions asymptotically approach such traveling wave solutions. In a traveling wave in one spatial dimension, the wave retains the same shape as it moves along the medium. Since the above model may asymptotically approach such a traveling wave, it is of interest to look for such a solution.

That is, let us focus attention far away from the colonization origin, where on the scales of interest planes of similar behavior should make the process essentially one-dimensional. And let us consider the possibility that for $r \gg x$ the main functions f (including P, Q, x, T, v, h) can be written $f(r,t) = f(r - \omega(t - t_0)) = f(e)$, where ω is the velocity of the traveling wave, $e(r,t) = r - \omega(t - t_0)$ is the extremity of a space-time event relative to the traveling wave center. Thus, with only a slight abuse of notation, we can write $f_r = f'$ and $f_t = -\omega f'$ (where $f' = f'(e)$).

For a traveling wave, the flow equation (7) becomes

$$(1 + \Delta e')\bar{P}'(e) = P'(e)\, G(x(e), T(e); v(e), h(e))\ \bar{Q}(e), \tag{12}$$

where for any f we write $\Delta f = \bar{f} - f$, the difference between succeeding generations. The motion of succeeding generations relative to a traveling wave is $\Delta e = x - \omega T$, so $\Delta e > 0$ for a lineage that advances relative to the wave, and $\Delta e < 0$ for a lineage that falls behind the wave.

Assuming leading-edge colonists just keep up with the leading edge, then $0 = \Delta e = \tilde{x} - \omega\tilde{T}$ there, so we must have $\omega = w$, with the wave speed being equal to the fastest sustainable leading edge speed from equation 2. If there were another place e behind the leading edge where $\Delta e'(e) = 0$ and $Q(e) < 1$, then by equation 12 we would have $G(e) > 1$, and so w would not be the fastest sustainable leading edge speed at $Q = 1$. Thus there is no other place where $\Delta e' = 0$, and so behind the leading edge we either have $\Delta e < 0$ everywhere, or $\Delta e > 0$ everywhere.

Since in a traveling wave $\ln\bar{Q}_t = -w \ln\bar{Q}_r$, the strategy FOC with respect to x, T (equations 8, 9) imply

$$\Delta\left(\frac{\ln G_x}{\ln\tilde{G}_x}\right) = \Delta\left(\frac{\ln G_T}{\ln\tilde{G}_T}\right), \tag{13}$$

where $\ln\tilde{G}_x, \ln\tilde{G}_T$ are the leading edge values. Since the ratios whose change equation 13 considers are equal at the leading edge, and since equation 13 says these ratios change by the same amount, these ratios must remain equal for any colony which has some ancestor or descendant at the leading edge. Thus assuming that all colonies have such a leading edge ancestor or descendant, they must all satisfy the same equation 3 which the leading edge must satisfy, and so

$$-w = \frac{\ln G_T}{\ln G_x} = \frac{\ln U_t}{\ln U_r}. \tag{14}$$

Equation 14 implies $U(r,t) = U(e)$, so there is no discounting along the traveling wave.

Result 7 *Colonists have infinite patience regarding investments whose returns keep up with a traveling wave.*

Though our model has assumed fixed technology, this result may be relevant to future models which consider colonists attitudes toward investing in research.

Given $U(r,t) = U(e)$, the strategy equation (5) for a traveling wave implies

$$U(e) = G(x(e), T(e); v(e), h(e))\ \bar{Q}(e)\bar{U}(e),$$

and equation 8 implies $U' > 0$. Thus $\Delta e \geq 0$ implies $\bar{U} \geq U$, and hence $GQ \leq 1$, with a declining number of descendants everywhere. Since this is not consistent with any sensible boundary condition on the flow of colonists at $r = 0$, we must have instead $\Delta e \leq 0$ everywhere, and hence $\bar{U} \leq U$ and $GQ \geq 1$.

Result 8 *In a traveling wave, succeeding generations of colonies on average fall further behind the leading edge and increase in number.*

In a traveling wave, the SOC for the strategy equation (5) with respect to x, T can be written

$$\forall\alpha, \quad 0 \geq \frac{\partial^2 \ln U}{\partial(x+\alpha T)^2} = \frac{\partial^2 \ln G}{\partial(x+\alpha T)^2} + \frac{(1-\alpha w)^2}{1+\Delta e'} \ln U'', \tag{15}$$

where we expect $1 + \Delta e'$ to remain positive to avoid a singularity in the flow equation (12), and where

$$\ln U'' = \frac{\ln G_{TT} T'^2 - \ln G_{xx} x'^2}{x' + wT'} = \frac{b^2 \ln A'' + \ln R''(T'^2 - x'^2/v^2)}{x' + wT'}.$$

Since increasing congestion and equation 8 imply $\ln\bar{U}' \leq \ln U'$, we have

$$\ln U'' > 0 \tag{16}$$

on average over each range $[e - \Delta e(e), e]$. If $\ln U'' > 0$ holds at each point, and if the SOC equation 15 is satisfied, we must have $\ln G$ strictly convex, which implies $\ln A'' \geq 0 \geq \ln R''$.

Result 9 *In a traveling wave, seeds are only launched when oasis growth rates are slowing down, and seeds only land when mortality rates are rising.*

Thus to the extent that the earliest colonization efforts, near $r = t = 0$, can be thought of as roughly following traveling wave strategies, colonists should not leave the colonization origin (such as our solar system) in earnest until growth rates begin to slow down here.

If we substitute our expression for G (equation 1) into equation 14, we find

$$\frac{1}{w} = \frac{1}{v} + b\frac{\ln A'}{\ln R'}, \tag{17}$$

Assuming positive growth and mortality, $\ln R', \ln A' > 0$, equation 17 immediately implies that $v > w$.

Result 10 *In a traveling wave, seeds always fly faster than the wave speed.*

Let us define best growth $\tilde{G}(x,T) = \max_{v,h} G(x,T;v,h)$ and consider the trajectory of succeeding generations in x,T space. Starting with some generation at the leading edge, and so at the point $\tilde{x}, \tilde{T}$ on the $T = wx$ line, succeeding generations move away from this point while the gradient of $\ln\tilde{G}$ remains orthogonal to this line (by equation 14) and the magnitude of this gradient decreases with succeeding generations (by equations 8 and 9).

If we assume $\ln G$ is strictly locally convex in x,T, then so is $\ln\tilde{G}$, which implies that along the curve of fixed gradient direction, the gradient must be monotonic in $\ln\tilde{G}$. Thus growth G increases with succeeding generations.

Result 11 *In a traveling wave, each colony expects to delivers more seeds to its succeeding space-time region than its parent expected to deliver.*

Equation 17 implies that if there is a lower bound to mortality $\underline{b\ln A'}$ and an upper bound to seed speed $\hat{v}$ (such as the speed of light), then there is also a lower bound to oasis growth rates

$$\ln R' \geq \underline{\ln R'} = \underline{b\ln A'}\frac{\hat{v}w}{\hat{v}-w}.$$

Since $GQ > 1$ and $G = R/CA$, we have $Q > CA/R$, so using our assumptions $C \geq 1$ and $A \geq 1$, an upper bound on oases resources $\bar{R}$ implies a limit on congestion,

$$Q \geq \underline{Q} = 1/\bar{R}.$$

And since $P = 1$ implies $Q = 0$, there must also be an upper limit $P \leq \bar{P} < 1$ to the fraction of oases which are colonized.

Result 12 *With finite oases and a minimum mortality in a traveling wave, there is an upper limit to the fraction of oases which get colonized, and a lower limit to the oasis growth rate colonists will tolerate.*

If one fixes seed speed and hardness v, h, then equation 17, $\bar{s} \geq s$, and locally convex $\ln G$ imply $\bar{x} \leq x$, which implies that if $\ln G_v = 0$ initially, then for children $\ln\bar{G}_v \leq 0$. Thus if we fix only seed hardness h, allowing speed to vary, we must have decreasing speed $\bar{v} \leq v$, which given $C_v \geq 0$ implies $\bar{C} \leq C$, so that number of seeds $N = R/C$ must increase. Equation 17 further implies that end-of-flight mortality $b\ln A' = (\ln A(bx))_x$ must decrease.

Result 13 *In a traveling wave with fixed seed hardness, each generation sends out more seeds at a slower speed, which have a lower mortality rate at the end of their flight.*

7 Smooth Traveling Wave In a Linear Model

If colonist strategies x, T, v, h do not change much from generation to generation, one might hope to approximate this model by a local differential equation.

For a traveling wave, equation 8 implies

$$w \ln\bar{Q} = \ln R' - \ln\bar{R}' = -\int_s^{\bar{s}} \ln R''(s) ds \approx -\ln R'' \Delta s,$$

the approximation being accurate when $\ln R''' \ll 2 \ln R''/\Delta s$. Similarly, when $\ln A''' \ll 2 \ln A''/b\Delta x$, and we fix seed speed and hardness v, h, then we have $(1 - w/v) \ln\bar{Q}' \approx -b \ln A'' \Delta x$. These approximations are exactly correct for *linear mortality & growth*, where $\ln R''' = \ln A''' = 0$.

These approximations describe expressions for $\Delta s, \Delta x$ in terms of $\ln\bar{Q}'$ and $\ln R'', \ln A''$, which given $\Delta e = (1 - w/v)x - ws$ imply

$$\frac{-\Delta(\Delta e)}{\ln\bar{Q}'} = K \equiv \frac{w^2}{-\ln R''} + \frac{(1 - w/v)^2}{b^2 \ln A''}.$$

We can further approximate $-K \ln\bar{Q}' = \Delta(\Delta e) \approx \Delta e' \Delta e = \frac{1}{2}((\Delta e)^2)'$ when

$$\Delta e'' \ll 2\Delta e'/\Delta e. \tag{18}$$

If we assume linear mortality & growth, then K is constant. Using the leading edge boundary condition that $\bar{Q} = 1$ at $\Delta e = 0$, i.e., that colonists at the leading edge tend to stay there, we then get

$$\bar{Q} = \exp(-\frac{(\Delta e)^2}{2K}), \tag{19}$$

an explicit Guassian functional relation between $\bar{Q}$ and Δe. And equation 14 can be rewritten

$$[-\ln R''(\frac{1}{w} - \frac{1}{v})]s + [b^2 \ln A'']x = [\ln R'(0)(\frac{1}{w} - \frac{1}{v}) - b \ln A'(0)], \tag{20}$$

where all bracketed quantities are constants. Equations 19 and 20, together with

$$\Delta e = (1 - w/v)x - ws \tag{21}$$

and the boundary values $\tilde{x}, \tilde{s}$, now completely describe the relationship between all parameters $\bar{Q}, \Delta e, x, s$ (assuming fixed v, h) in a smooth traveling wave for linear mortality & growth.

The only thing we haven't described is how any one of these parameters varies explicitly with position e, where a colony sits relative to the leading edge. That requires making use of the population flow equation 12.

Result 14 *For fixed seed hardness and speed, and linear mortality & growth in a slowly-varying (equation 18) traveling wave, equations 19, 20, 21 give explicit relationships between how long colonists wait at oases s, how far they fly x, how much each generation falls behind Δe, and the chance seeds land at an open oasis $\bar{Q}$.*

8 Modeling Congestion

We have defined congestion Q in terms of $q(p, p_t)$, which gives the probability of landing at an unoccupied oasis when a fraction p of oases are occupied, when occupancy is increasing at a rate p_t. We now consider an explicit binary-signal model for q.

One expects colonies and seeds to study possible target oases, and perhaps communicate with other nearby seeds in flight regarding landing intentions. Assume that at the time they must commit to a choice of target, seeds consider a binary signal regarding whether an oasis is open ($\circ$) or closed ($\bullet$), i.e., available to colonize or will have been taken already.

Let $q_\circ$ be the conditional probability of getting an open signal given an open oasis, and let $q_\bullet$ be the conditional probability of a closed signal given a closed oasis. A seed which then randomly chooses among open-signal oases then has a chance Q to land at an open oasis, given by

$$\frac{1-Q}{Q} = \epsilon \frac{P}{1-P},$$

where error $\epsilon = (1 - q_\bullet)/q_\circ \leq 1$. This satisfies $Q \geq 1 - P$ and $P = 1$ implies $Q = 0$, as we have assumed.

While our model has so far not allowed for a choice of error ϵ, if such a choice were allowed, an optimal choice would

$$\min_\epsilon C \left(1 + \epsilon \frac{P}{1-P}\right),$$

which for $C_\epsilon < 0$ implies that C increases with P.

Result 15 *The fewer open oases remain, the more seeds spend to discern which oases remain open.*

If the difficulty of detecting colonized oases depends on how long those oases have been colonized, then the expression for ϵ is modified. Assume that a recently colonized oasis, roughly less than τ old, has different signal correlations $\hat{q}_\circ$, $\hat{q}_\bullet$. Then with a fraction $D \approx \tau p_t$ of recently-colonized oases, we have

$$\epsilon = \frac{(1-q_\bullet)(1-D) + D(1-\hat{q}_\bullet)}{q_\circ(1-D) + D\hat{q}_\circ}.$$

Note that with this form, if P_t is not monotonic in P, then Q need not be monotonic in P, and so increasing population $P_t \geq 0$ need not imply increasing congestion $Q_t \leq 0$.

9 Risk-Aversion

The form of the strategy equation (5) assumes risk-neutral colonists. It seems reasonable to assume that the risks of seed mortality are largely independent across widely separated seeds, and that there are large numbers of widely separated seeds with copies of any one "gene" coding for colonist behavior. Given these assumptions, any plausible level of risk aversion for a gene should translate to near risk-neutrality regarding the risks to any one seed (Blume & Easley, 1992). Thus risk-neutrality should be a reasonable approximation, at least regarding large numbers of widely separated seeds.

Risk neutrality is a more suspect approximation regarding risks which are strongly correlated over large regions, such as the risk of a gamma ray burst, or that this colonization wave will hit an opposing wave coming from another source. These possibilities have not been modeled in this paper.

Note also that risk-neutrality seems a bad approximation when mortality is exactly constant. To see this, consider any strategy, starting at r, t, of waiting s_1 and then going x_1, followed by waiting s_2 and then going x_2, in both cases using the same v, h. For risk-neutrality and constant mortality, this sequence is dominated by the strategy of waiting s_1, going $x_1 + x_2$, waiting s_2 and then going $x = 0$, at least if by waiting one encounters less congestion, as in

$$Q(r + x_1, t + s_1 + x_1/v) < Q(r + x_1 + x_2, t + s_1 + (x_1 + x_2)/v).$$

The waiting strategy has exactly the same total growth at oases, R_1 and R_2, and given constant morality has the same total mortality between oases $A_1 A_2$, but has better congestion Q. That is, a seed would always prefer to fly farther to land rather than to suffer more congestion from landing earlier. But if all seeds do this, none would ever land, a nonsensical result.

Result 16 *Risk-neutrality is a bad approximation when mortality is exactly constant.*

10 Noisy Landings

Note also that the form of equation 5 neglects to explicitly include noise regarding landing events. A form which included noise would look something like

$$\max_{x,T} \int G(\ldots)\, Q(\ldots)\, U(r + x + \epsilon_x, t + T + \epsilon_T) dF(\epsilon_x, \epsilon_T; x, T).$$

The form we have used is, however, a reasonable approximation to this form when noise is small. To see this, consider maximizing some $V(z)$ when z has noise distributed by $F(\epsilon)$, and assuming $0 = \int \epsilon dF(\epsilon)$ and $0 = V'''' = V''''' \ldots$. Then the error from ignoring noise is

$$\begin{aligned}
\Delta z &= \text{argmax}_z \int V(z+\epsilon) dF(\epsilon) - \text{argmax}_z V(z) \\
&= S(1 - \sqrt{1 - (\sigma/S)^2}) \quad \text{for } \sigma \leq S \\
&\approx \frac{\sigma}{S}\left(\frac{\sigma}{2}\right) \quad \text{for } \sigma^2 \ll S^2,
\end{aligned}$$

where $\sigma^2 = \int \epsilon^2 dF(\epsilon)$ is the standard deviation of noise, and $S = -V''/V'''$ evaluated at $\text{argmax}_z V(z)$ is the distance from this point where the SOC, $V'' \leq 0$, no longer holds. Thus for $\sigma \ll S$, we have $\Delta z \ll \sigma$, a small correction indeed.

Result 17 *Our analysis seems robust to substantial noise in where and when seeds actually land.*

11 Seed Diversity

We have so far assumed that all seeds launched from an oasis are launched at the same time. Let us now consider allowing more diversity in seek launching dates.

Think of an oasis as made of a variety of resources, some of which permit faster growth than others. Some asteroids, for example, may combine good element compositions with good access to starlight, while others may have combine poor starlight access with deficits of certain key elements. We expect colonies to begin growing in the best resources, and to switch to less desirable resources as the best resources become depleted.

Assume that a colony can at any time be described by it's working capital $K(s)$ and the resources $R(s)$ that it has so far consumed, and that the colony begins with $K(0) = R(0) = 1$. The colony can consume resources at a rate $R' = Kg(R)$, where $g(R)$ is the capital growth rate possible after R resources have been consumed. The colony output R' can be devoted to either further capital growth K' or to sending out seeds. Previously accumulated capital K can also be devoted to sending seeds.

If the value of a launched seed declines with the launch date according to $V(s)$, the colony should thus want to choose $K(s)$ to maximize

$$\int_0^\infty \left[K(s)g(R(s)) - K'(s)\right] V(s)ds \tag{22}$$

subject to $R(0) = K(0) = 1$, $K \geq 0$, and $K'(s) \leq K(s)g(R(s)) = R'(s)$. Before considering this problem directly, however, let us consider the related problem where growth rates are fixed with time: maximize

$$\int_0^\infty \left[K(s)g(s) - K'(s)\right] V(s)ds \tag{23}$$

subject to $K(0) = 1$, $K \geq 0$, and $K'(s) \leq K(s)g(s)$.

The Euler-Lagrange first order condition for problem 23 is

$$g(s) = \gamma(s),$$

where we have defined $\gamma(s) = -\ln V'(s)$, a local discount rate. This equation says that for interior optima, the local growth rate must equal the local discount rate. Thus if $g(0) > -\gamma(0)$, we must initially have a boundary solution where $K'(s) = K(s)g(s)$, with all resources devoted to capital accumulation. If feasible growth rates decline relative to discount rates, until a time t^* when such rates become equal, with $g(s^*) = \gamma(s^*)$, then this all-capital boundary solution will continue until s^*, at which time capital can be spent on sending out seeds. If $g'(s) < \gamma'(s)$ after s^* as well, then there must be a boundary solution after s^*, with $K = 0$. Thus all capital is spent on seeds at s^*.

If we now consider $g(R(s))$ instead of $g(s)$, we can ask whether this solution to problem 23 also describes a solution to problem 22. That is, let us define $\bar{R}$ as the maximal capital accumulation growth path which solves $\bar{R}'(s) = \bar{R}(s)g(\bar{R}(s))$ and $\bar{R}(0) = 1$, let us analogously assume that $g(1) > \gamma(0)$, that $g'(\bar{R}(s))\bar{R}'(s) \leq 0 \leq \gamma'(s)$, and that for some s^* we have $g(\bar{R}(s^*)) = \gamma(s^*)$. Now consider whether it is optimal to accumulate capital along $K(s) = \bar{R}(s)$, and then spend it all sending seeds at s^*.

There are two possible deviations to consider: sending seeds out before s^* or afterward. If some capital is spent sending seeds early, at a time $s_1 < s^*$, this deviation is a net win only if the added gain from getting an earlier seed outweighs the loss from delaying when the other seeds can be launched. That is, if

$$V(s_1) - V(s^*) > V'(s^*)\bar{R}(s^*) \int_{\bar{R}(s_1)}^{\bar{R}(s^*)} \frac{-dR}{g(R)R^2},$$

where the integral describes how the ideal seed sending time s^* varies with earlier capital $K(s_1)$. This condition is impossible to meet given our assumption that growth rates are faster than discount rates over this period. An intuition for this is aided by noting that with constant growth and discount rates, i.e., $g(R) = g$ and $\gamma(s) = \gamma$, the deviation win condition becomes

$$(e^{\gamma\Delta s} - 1)/\gamma > (e^{g\Delta s} - 1)/g$$

for $\Delta s = s^* - s_1$, which contradicts our assumption that $\gamma \leq g$ in this region.

The other deviation to consider is to save and not spend some capital at s^*, growing it instead to send a seed at some later date. Our assumption of increasing discount rates, $\gamma' > 0$, ensures that this capital will grow at a rate $g(R(s)) \leq g(\bar{R}(s^*)) \leq \gamma(s)$. Hence the growth rate would be less than the local discount rate, giving a net loss for this deviation.

Our traveling wave analysis, however, suggested decreasing, not increasing, discount rates (see equation 16). And in this situation it *is* optimal to leave some capital behind, and to continue to save enough capital to maintain the relation $g(R(s)) = \gamma(s)$. The optimal capital fraction, however, is

$$\frac{K}{R} = \frac{\gamma'(s)/\gamma}{R\, g'(R)} = \frac{d\ln\gamma}{ds} \Bigg/ \frac{dg}{d\ln R},$$

which is very small when discount rates vary slowly with time and growth rates vary substantially with resources R. This suggests that almost all seeds are sent in the main wave at s^*, making it reasonable approximation to assume in our main analysis that all seeds are sent at the same time.

Result 18 *When local growth rates decline as more resources are consumed, and when local discount rates vary slowly, almost all seeds are launched at the same time.*

Even assuming that the small-fraction of "left-behind" colonists has a negligible effect on the rest of our analysis, however, such colonists are of particular interest in their own right. After all, most of the volume of the colonized universe should eventually be far behind the leading edge, and this volume may be dominated by such left-behind colonists. Besides strategic reasons, colonists may be left behind due to random factors, or due to being embodied in fixed sunk capital such as seed launchers.

12 Oasis Diversity

We have so far required all oases to have the same resource function $R(s)$, but most of our results generalize to the case where resources $R_\beta(s)$, and hence growth G_β depend on the oasis type β.

Assume that seeds have no trouble distinguishing between oases of different types, though they may still have trouble determining if an oasis is colonized. With oasis diversity, leading edge colonists will then seek out the most favorable oasis type, and so the leading edge maximum speed equation (2) will also maximize over β, choosing some best value $\tilde{\beta}$. Our previous results about the leading edge apply regarding this type of oasis $\tilde{\beta}$.

If seed costs do not depend on the type of oasis they are to land at, then if landings are being attempted at more than one type of oasis in any space-time region, colonists must be indifferent between landing at these oasis types. Assume that landings are always being attempted at the best type $\tilde{\beta}$ if they are being attempted at any type, and for any f let the unsubscripted version be for that best type, as in $f = f_{\tilde{\beta}}$. Then

$$Q_\beta(r,t)\, U_\beta(r,t) = Q(r,t)\, U(r,t).$$

must be satisfied for all β where $P_t(\beta) > 0$. An oasis type for which seed landings are locally being attempted for the first time must have $Q_\beta = 1$, and so all else equal, the lower $Q = Q_\beta$ is, the worse a U_β/U ratio can be tolerated for a type β.

Result 19 *All else equal, greater congestion induces seeds to attempt landings at more oasis types.*

An analogue to the strategy equation (5) must apply for each colonized oasis type,

$$U_\beta(r,t) = \max_{x,T,v,h} G_\beta(x,T;v,h)\, Q(r+x,\, t+T)\, U(r+x,\, t+T).$$

and so the same form of FOC, equations 8 and 9, must be satisfied for all such oasis types. Thus all oases who try to land seeds at the same space-time region must have the same values of $\ln G_x$ and $\ln G_T$, hence the same value of $\ln R'_\beta$, and hence exactly the same equations determining x, v, h.

Result 20 *Even with diverse oasis types, all seeds launched from a given space-time region have the same speed and hardness, and target the same destination space-time region.*

Consider the situation where in some small region some oases are launching seeds now while other oases are still waiting to launch, and still others have already launched. All the launching oases have the same cutoff growth rate $\ln\hat{R}'$. When local growth rates slow with time, $\ln R''_\beta < 0$, then if another nearby colony is waiting longer, it must be because it is growing faster than this cutoff rate, $\ln R'_\beta > \ln \hat{R}'$, while a colony that has already launched must have expected to have a lower-than-cutoff growth rate had it not launched.

Thinking in terms of multiple oasis types can help clarify intuitions regarding the behavior of left-behind colonists. A post-exodus oasis can just be thought of as an already-colonized oasis of a new type β. If the exodus process decreases local growth rates, then left-behind colonists would want to launch more seeds as soon as possible, rather than staying to grow further. (In the model in the previous section (11), exodus left growth rates unchanged.)

Result 21 *All local oases with some cutoff growth rate launch seeds, while faster growing oases wait longer. If seed launching decreases local growth rates, left-behinds want to leave A.S.A.P.*

13 Observational Implications

We have tried in this paper to model colonists who descend from leading edge colonists, and who have retained their ancestor's preferences, all far away from the origin of an expanding colonization wave. One possible application of this model is to understand our descendants' future, assuming that they eventually engage in interstellar colonization, that light speed remains a fundamental limit to travel speeds, and that alien ETI are sufficiently distant that there is time for selection effects to determine frontier behavior.

Another possible application of this model is to try to reconcile the "Great Silence" we see in astrophysics with a hypothesized previous wave of ETI colonization that might have long ago passed this way. How consistent are the implications of this model with astrophysical observation?

13.1 Puzzling Observations

No known astrophysical phenomena seem puzzling if one assumes that any one star system is extremely unlikely to give birth to an expanding wave of interstellar colonization. If simple life is extremely unlikely to develop in any one system, or if simple life were extremely unlikely to generate a civilization powerful enough to begin interstellar colonization, then one can believe that over the ten billion plus years our universe has existed, none of the over 10^{20} star systems we can now see were the origin of a colonization wave that passed this way. In this section, however, we focus on reconciling our astrophysical observations with more optimistic estimates of the chances for originating powerful civilizations.

The lack of alien visitors here on Earth now is striking (Brin, 1983), but upon reflection is not terribly surprising. Even if the vast majority of star systems in our galaxy have been colonized by ETI, there are many reasons to expect a few exceptions. Our star system might be part of an isolated zoo (Ball, 1973), for example, or in an empty border zone (Turner, 1985).

Our failure to so far detect strong radio communications between ETIs may also not be terribly surprising. After all, we have sampled only a very small fraction of the space of not-obviously unreasonable communication system parameters. Our failure to detect communications directed at us may perhaps be more surprising, if we assume Earth is of interest to ETIs. For example, when we looked at 209 nearby stars like our sun, we failed to hear anything like the equivalent of Earth's currently most powerful transmitter (Arecibo Radar) talking to us. At the ten nearest such stars, we failed to hear any transmitter 500 times weaker (Dreher & Cullers, 1997).

On reflection, perhaps the most puzzling astrophysical observation is the vast quantities of apparently unused resources out there. If one thought most stars were colonized, and if one expected mature stellar civilizations to make use of a substantial fraction of the resources near their star, then one might be surprised to learn that a survey of 230 observed nearby stars like our sun found that less than one percent of the starlight of each is being intercepted by local matter (Jugaku & Nishimura, 1997) (at least matter that reradiates near 300K). This seems a striking contrast to some predictions, such as by Barrow and Tipler (Barrow & Tipler, 1986), of a complete exhaustion of material resources in the wake of a colonization wave.

The great success of stellar physics also indicates that very few stars have been much modified by local civilizations, such as to extract mass. And we observe vast quantities of apparently unused material in molecular and dust clouds. Such apparently virgin mass may seem puzzling given than even now we can sketch out engineering projects which might make use of such resources (Criswell, 1985). We can also see that civilizations surrounding nearby stars dump very little nuclear waste into their stars (Whitmire & Wright, 1980), and we can place limits on the use of powerful radars (Tarter, 1997) and certain forms of starship drives (Drake & Sobel, 1992).

13.2 Reconciling the Puzzles

The model in this paper suggests a way to reconcile the hypothesis of a colonization wave having once passed this way with observations of apparently low activity levels and resource utilization nearby. The model implies low local activity levels, as there should be few colonists left far behind the colonization frontier. The model also implies that any such colonists would be in a rush to try to catch up with the frontier, which does not offer any obvious motivation to broadcast signals at Earth. And the model accounts predicts a small residue of untouched oases, such as our solar system seems to be.

The model might also explain observations of unused starlight, stellar mass, and other mass sources, if these resources cannot support rapid growth. We mainly need to assume that large engineering projects, such as required to collecting most of the light of a star or to extract much of its mass, just take too long to produce an acceptable rate of return. This might explain both the unused resources and the absence of large visible engineering projects.

While the model can explain these previously puzzling observations, it also predicts a number of new phenomena that we have not yet observed, nor failed to observe. Future observations which look more carefully for such phenomena will thus offer opportunities to test this model.

First, the model predicts that a "wall" of fresh colonization frontier activity should be observable at some distance, though that distance could be very far if the frontier passed this way a long time ago. Given a specification of oasis growth and in-flight mortality functions, the model even predicts how thick this wall should be, and how fast it would be moving. How visible the wall should be, however, depends on how much radiation is produced by fresh colonization activities.

Second, the model predicts an *absence* of local resources which can support rapid growth. If for any given resource we can estimate the most rapid possible growth rate possible using the best technological designs, then we should see a cutoff in the distribution of such growth rates, across the types of resources observed. This value of this cutoff should be consistent with a fuller model of the colonization process, including a wave speed and rates of in-flight mortality. And models of the physical processes which produce the distribution of resources available should predict the generation of fast-growth resources which we fail to observe nearby.

Third, we should find garbage and remains where there were once resources supporting rapid growth. Up close we might even find the remains of any fixed sunk costs such as mines and seed launchers, as colonists in this model have no particular reason to want to cover their tracks. The better colonists were at recycling capital, however, the less such garbage should remain.

Fourth, we may find the remains of seeds which suffered from in-flight mortality, as well as any exhaust from in-flight propulsion and navigation. Of course it is possible that most propulsion was produced in the launching system, and that most of the remains of failed seeds continued on until colliding with its intended destination system. But some residues may remain to be detected.

13.3 A Further Complication

A further complication is that this model would seem to allow for several colonization waves, at slower and slower speeds w. After the first colonization wave has passed through a region, another slower wave might pass through, using oases and their remainders that grow too slowly to interest the first wave. Colonists in slower waves would have substantially different preferences and behavior from first-wave colonists. They might originate from the same colonization origin as the first wave, or from large preference mutations in colonists during first wave.

If the first wave did not collide with waves from other sources, if the second wave came from the same origin as the first, and if it had a speed substantially less than 79% of the speed of the first wave, then the second wave would have reached substantially less than half of the volume of the first wave, and predictions which ignored later waves might remain reasonable. In other cases, a more complex analysis seems required.

In summary, this model allows our solar system to have been passed by a colonization wave, and yet be an untouched oasis in a very large region containing much unused resources, and little advanced activity or large artifacts. But the model also predicts a colonization wave observable at some distance, an absence of physically-expected fast-growth resources nearby, possibly replaced by a few garbage artifacts, and perhaps some remains from in-flight seed mortality and propulsion. More careful astrophysical analysis seems required to more directly confront these predictions with observations. And a more careful analysis of multiple colonizations waves may be in order.

14 Conclusion

It seems desirable to try to replace astronomer's intuitive social theories with more formal economic theory in directing the empirical search for ETI. In that spirit, this paper takes a few commonly considered assumptions, and given those assumptions considers the interstellar colonization behavior of sophisticated actors with many behavioral choices. The hope is that this theoretical analysis may be both general enough to hope to apply to the extreme circumstance of ETI societies, and formal enough to allow a clearer identification of which assumptions lead to which conclusions.

Far enough away from the origin of an expanding wave of interstellar colonization, and in the absence of property rights in virgin oases, a selection effect should make leading edge colonists primarily value whatever it takes to stay at the leading edge. Metaphorically, a "wildfire" would burn through the "cosmic commons," turning most easy kindling into a residue of charcoal and ash, but leaving slow-burning trunks mostly untouched. After the fire has past, the rare untouched sapling would find itself in a large quiet region with few competitors.

That is, to the extent that descendants of such colonists who fall behind the leading edge retain their ancestor's preferences, such descendants should focus on pursuing the colonization wave as quickly as possible, rather than staying behind to build stable long-lasting civilizations. While such colonists would adjust their strategies to deal with increased congestion, there are limits to have far their strategies vary from leading-edge strategies. Thus our observations of apparently being a lone oasis among a large region almost devoid of large scale advanced activity may be consistent with a colonization wave having once passed this way.

This analysis has neglected to consider many important issues, which will hopefully be addressed by future research. Perhaps most seriously, this analysis has not much considered the possibly of slower waves of colonists whose preferences differ substantially from those selected-for at the leading edge of the first wave. And we have not very carefully considered non-uniformities in the inter-oasis medium.

Regarding the behavior of colonists left far behind the leading edge, it seems important to explicitly model attitudes toward risk, the possibility of property rights and other forms of coordination among colonies, and possible selection-induced drift in colonist preferences.

The other end of the colonization process also seems worthy of more attention. Assuming one is at the beginning of a colonization process, and that one wants one's descendants to stay competitive with the colonization wave, what strategies would that imply? When should they leave, for example? At this end of the process, it seems especially important to consider risk aversion and to allow for technological change.

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15 Glossary

A Number of seeds which must be launched to expect one to survive the flight.

b Parameter controlling the rate at which seeds are destroyed in flight.

β Type of oasis.

C Cost to launch one more seed, assuming the fixed cost has been paid.

δ Time delay required for a seed to stop and begin to grow at an oasis.

D The fraction of oases which have been colonized "recently".

e Position of a space-time event relative to a traveling wave.

ϵ The error rate in signals regarding whether oases are still available.

F Fixed cost required to support launching of seeds.

G Expected number of seeds a single grown seed delivers to the next space-time region.

h Hardness of seed.

n Number density of oases in local region of space.

N Number of seeds launched from an oasis.

P Fraction of local oasis that have already been colonized.

$q_\circ(q_\bullet)$ The conditional probability of seeing a truthful signal indicating an open (closed) oasis.

Q Probability that a seed will land at an already-colonized oasis.

r Distance from origin of colonization process.

R Resources available to colony at an oasis, net of fixed launching cost.

$\tilde{R}$ Total resources available to colony at an oasis.

s Time duration a colony has been at an oasis, including delay for seed to stop at oasis.

t Time since the start of colonization process

T Time duration between seed generations, from seed landing to seed landing.

U Utility of having a seed land at an unoccupied oasis in some space-time region.

v Velocity of seed in flight.

w Average maximum sustainable speed of colonization

ω Average speed of a traveling wave of colonization.

x Distance a seed travels from where it was launched to where it lands.

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f Some arbitrary function of space and time.

$\ln f$ A new function of space time equal in value to the natural logarithm of the old one.

f_r The partial derivative of f with respect to location r.

f_t The partial derivative of f with respect to time r.

f' The partial derivative of f relative to the wave position along a traveling wave.

Δf The difference between f for seeds landing here now, and f for the next generation.